UNIT-1 MATHEMATICAL LOGIC

Introduction:

It develops mathematical thinking, problem solving capabilities, it place an important role in the field of computer science in the areas like compiler design, Data bases, computer security or automat Theory, Computer representation of Discrete Structures many problems can be solved using Discrete Mathematics.

Statement or Proposition:

Proposition or statement is a Declarative sentence with true or false.

Eg:- 1. Sun rises in the east. (True Statement) (Proposition).

2. $2+5=7$ (True Statement)

$2+4=9$ (False Statement)

$x+4>1$ is not declarative Sentence. (neither True ~~of~~ or False)

Generally prepositions are represented by the letter "p, q, r".

Types of Propositions:

They are classified into two types:

- Automic or simple or primary or premetive.
- Compound Proposition.

Atomic Proposition :

An atomic Proposition is a proposition which can't be divided further i.e, it doesn't contain any connectives. $\vee, \wedge, \rightarrow, \leftrightarrow$.

Eg:- Mumbai is capital of India.

we can't divide the proposition into several parts.

Compound Proposition :

It is a combination of connectives such as $\vee, \wedge, \rightarrow$ or $\leftrightarrow$.

Logical Connectives:

Connectives are mainly useful inorder to combine two or more prepositions.

mainly we have 5 connectives :

- Conjunction ($\wedge$)
- Disjunction ($\vee$)
- Negation ($\sim$)
- Conditional ($\rightarrow$)
- Biconditional ($\leftrightarrow$)

Conjunction: If p and q are two statements then the conjunction of p, q is denoted by $p \wedge q$. Defined as $p \wedge q$ is true.

p	q	$p \wedge q$	$p \vee q$	$\sim p$	$\sim q$	$p \rightarrow q$	$p \leftrightarrow q$
T	T	T	T	F	F	T	T
T	F	F	T	F	T	F	F
F	T	F	T	T	F	T	F
F	F	F	F	T	T	T	T

A variable is called a boolian variable if its value is either True or False.

Truth Table of Bit Operations : OR ($\vee$), AND ($\wedge$), XOR $\oplus$

x	y	$x \wedge y$	$x \vee y$	$x \oplus y$
1	1	1	1	0
1	0	0	1	1
0	1	0	1	1
0	0	0	0	0

1 - True
0 - False

Prepositional Equivalance :

· Tautology : A compound statement which is always True is called Tautology.

Example: i) $p \vee \sim p$

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

$\therefore$ $p \vee \sim p$ is Tautology.

ii) $\sim (p \wedge q) \vee q$

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim(p \wedge q) \vee q$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

$\therefore$ $\sim(p \wedge q) \vee q$ is Tautology.

· **Controduction**: A ~~component~~ compound statement which is always False.

Example: $p \wedge (q \wedge \sim p)$

p	q	$\sim p$	$q \wedge \sim p$	$p \wedge (q \wedge \sim p)$
T	T	F	F	F
T	F	F	F	F
F	T	T	T	F
F	F	T	F	F

$\therefore p \wedge (q \wedge \sim p)$ is controdiction.

· **Contingency**: A compound statement which is neither a Tautology nor controdiction.

Example: i) $p \rightarrow q$ ii) $p \leftrightarrow q$

p	q	$p \rightarrow q$	$p \leftrightarrow q$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	T

$\therefore p \rightarrow q$ and $p \leftrightarrow q$ are contingency.

Converse, Contro positive and Inverse Statements:

For $p \rightarrow q$

converse : $q \rightarrow p$

· $q \rightarrow p$ is called converse of $p \rightarrow q$

contro positive : $\sim q \rightarrow \sim p$

· $\sim q \rightarrow \sim p$ is called the contro positive of $p \rightarrow q$

inverse : $\sim p \rightarrow \sim q$

· $\sim p \rightarrow \sim q$ is called the inverse of $p \rightarrow q$.

2. Write the converse, contro positive and ~~to~~ inverse of the following statements.

i) The home team wins then it is raining.

here, q: home town wins
p: it is raining.

converse: If howe ~~tom~~ town wins then, it is raining.

contro positive: If home town doesn't win then, it is not raining.

inverse: If it is not raining then, ~~the~~ home town doesn't win.

ii) If it rains today then I'll stay at home.

here, q: it rains today
p: I'll stay at home.

converse: If it rains today then, I'll stay at home.

contro positive: If it doesn't rain today. Then, I'll not stay at home.

inverse: If I'll not stay at home then it'll not rain To day.

iii) If you recieve 'A' grade in Discrete Mathematics then you'll be awarded scholarship.

iv) If exercise is good for health then, I'll go to to the park.
Some where.

11/03/2024:

Negation:

- $\sim(p \wedge q) = \sim p \vee \sim q$
- $\sim(p \vee q) = \sim p \wedge \sim q$
- $p \rightarrow q = \sim p \vee q$
- $\sim(p \rightarrow q) = \sim(\sim p \vee q)$
  $= \sim(\sim p) \wedge \sim q$
  $= p \wedge \sim q$
- $p \leftrightarrow q = (\sim p \vee q) \wedge (\sim q \vee p)$
- $\sim(p \leftrightarrow q) = \sim\left[(\sim p \vee q) \wedge (\sim q \vee p)\right]$
  $= \sim(\sim p \vee q) \vee \sim(\sim q \vee p)$
  $= (p \wedge \sim q) \vee (q \wedge \sim p)$

Statement to Symbolic Form:

1. Let, p: He is tall
   q: He is handsom

i) He is tall and handsom : $p \wedge q$.

ii) He is tall but not handsom : $p \wedge \sim q$

iii) It is false that that he is not tall or handsom
$\sim(\sim p \vee q)$.

iv) He is neither tall nor handsom : $\sim p \wedge \sim q$

v) He is not tall or he is not tall and handsome : $p \vee (\sim p \wedge q)$

vi) It is not true that he is not tall or not hansome:

$\sim(\sim p \vee \sim q)$

(2) You are not allowed to watch adult movies if your age is less than 18 years or you have no age proof.

Let p: your allowed to watch adult movies.

q: less than 18 years

r: you have age proof.

$\sim p \rightarrow (q \vee \sim r)$

(3) If either Ram takes C++ or Kumar takes Pastal then Lata will take Lotus.

Prepositions

p: Ram takes C++

q: Kumar takes Pastal

r: Lata will take Lotus.

Logical Connective:

$(p \vee q) \rightarrow r$

(4) a: It is raining then there are clouds in the sky.

b: If it is not raining then the sun is not shining and there are clouds in the sky.

c: Sun is Shining if and only if It is not raining

Propositions: p: It is raining

q: there are clouds in the sky.

r: the sun is shining

Therefore, a: $p \rightarrow q$

b: $\sim p \rightarrow (\sim r \wedge q)$

c: $r \leftrightarrow \sim p$

H.W

5. p: Neru is rich.

q: Neru is happy.

write in symbol form a) Neru is poor but happy.

b) Neru is rich or unhappy.

c) Neru is neither rich nor happy.

d) It is necessary for Neru to be poor in order to be happy.

e) Neru ~~is~~ to be poor is to be ~~no~~ unhappy.

f) Neru is rich or both poor and ~~or~~ unhappy.

14/3/24

Prove that $p \vee (\sim p \wedge \sim q) \equiv p \vee \sim q$

LHS: $p \vee (\sim p \wedge \sim q)$

$= (p \vee \sim p) \wedge (p \vee \sim q)$

$= T \wedge (p \vee \sim q)$ $\quad$ [$\because$ Destribution Law]

$= p \vee \sim q$ $\quad$ [$\because$ Negation Law]

$=$ RHS

Hence Proved.

$p \wedge (q \vee \sim p) \equiv p \wedge q$

LHS: $p \wedge (q \vee \sim p)$

$= \cancel{(p \wedge q) \vee (p \wedge \sim p)}$ $\quad$ Destribution Law

$= \cancel{p \wedge q \vee T}$

$= (p \wedge q) \vee (p \wedge \sim p)$ $\quad$ ($\because$ Destribution law)

$(p \wedge q) \vee F$ $\quad$ ($\because$ Negation Law)

$= p \wedge q$ $\quad$ ($\because$ Identity Law)

LHS $=$ RHS

$(p \vee q) \wedge \sim(\sim p \wedge q) \equiv p.$

LHS $\equiv (p \vee q) \wedge \sim(\sim p \wedge q)$

$\equiv (p \vee q) \wedge (\sim(\sim p) \vee \sim q)$

$\equiv (p \vee q) \wedge (p \vee \sim q)$ (double Negation)

$\equiv p \vee [q \wedge \sim q]$ ($\because$ Negation Law)

$\equiv p \vee F$

$\equiv p$ [$\because$ Identity Law]

LHS $\equiv$ RHS

$(p \to q) \wedge (q \wedge (r \vee \sim q)) \equiv \sim(p \vee q)$

LHS $= (p \to q) \wedge (q \wedge (r \vee \sim q))$

$\equiv (p \to q) \wedge [\sim q \wedge (r \vee \sim q)]$

($\because$ absorption Law)

$\equiv (p \to q) \wedge \sim q$

$\equiv (\sim p \vee q) \wedge \sim q$ ($\because$ Commutative Law)

$\equiv (\sim p \wedge \sim q) \vee (q \wedge \sim q)$ ($\because$ Distribution Law)

$\equiv (\sim p \wedge \sim q) \vee F$ ($\because$ Identity Law)

$\equiv \sim p \wedge \sim q$

Hence Proved.

$\equiv \sim(p \vee q)$

$\equiv$ RHS

$$\sim[(p \vee \sim q) \rightarrow (p \wedge \sim q)] \equiv (p \vee \sim q) \wedge (\sim p \vee q)$$

$$LHS \equiv \sim[\sim(p \vee \sim q) \vee (p \wedge \sim q)]$$

$$\equiv [(p \vee \sim q) \wedge \sim(p \wedge \sim q)] \quad [\because \text{Negation Law}]$$

$$\equiv (p \vee \sim q) \wedge (\sim p \vee q)$$

$LHS = RHS$

~~Hainh~~ Hence Proved.

H.W

$$p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$$

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

$$\equiv (\sim p \vee q) \wedge (\sim q \vee p)$$

Normal Forms:

If we write the given statement formula interms of $\wedge, \vee, \sim$ then it is called normal / canonical form.

There are four Types of normal form:

- Disjunctive Normal Form
- Conjuctive Normal Form
- Principal Disjunctive
- Principal Conjuctine

Disjunctive Normal Form:

A logical expression is said to be disjunctive normal form if it is sum of elementory products i.e;

which is DNF ($\vee$)

$$DNF = \begin{pmatrix} \text{elementary} \\ \text{Product} \end{pmatrix} \vee \begin{pmatrix} \text{elementry} \\ \text{Product} \end{pmatrix} \vee \begin{pmatrix} \text{elementry} \\ \text{Product} \end{pmatrix}$$

$$= (p \wedge \sim q) \vee (q \wedge \sim p) \vee (p \wedge q).$$

Conjuctive Normal Form: CNF ($\wedge$)

$$CNF = \begin{pmatrix} \text{elementry} \\ \text{Sum} \end{pmatrix} \wedge \begin{pmatrix} \text{elementry} \\ \text{Sum} \end{pmatrix} \wedge \begin{pmatrix} \text{elementry} \\ \text{Sum} \end{pmatrix}$$

$$= (p \vee q) \wedge (p \vee \sim q) \wedge (\sim p \vee \sim q)$$

A logical expression is said to be Conjuctive normal form if it is product of elementry sum.

NOTE: · DNF and CNF are not unique.

PDNF and PCNF are unique.

Minterms: Given a number of variables, the products in which each variable or its negation but not both occurs only once. These are called Min Terms.

p	q	Min Terms ($\wedge$)
T	T	$p \wedge q$
T	F	$p \wedge \sim q$
F	T	$\sim p \wedge q$
F	F	$\sim p \wedge \sim q$

$\therefore$ Min Terms are $p \wedge q$, $p \wedge \sim q$, $\sim p \wedge q$, $\sim p \wedge \sim q$.

MaxTerms: Given a number of variables, the sum in which each variable or its negation but not both occurs only once. These are called Maxterms. (1)

p	q	Max terms ($\vee$)
T	T	$\sim p \vee \sim q$
T	F	$\sim p \vee q$
F	T	$p \vee \sim q$
F	F	$p \vee q$

$\therefore$ Maxterms are $\sim p \vee \sim q$, $\sim p \vee q$, $p \vee \sim q$, $p \vee q$.

15/12/24

Minterms and Maxterms for 3 propositions

p	q	r	Minterms ($\wedge$)	Maxterms ($\vee$)
T	T	T	$p \wedge q \wedge r$	$\sim p \vee \sim q \vee \sim r$
T	T	F	$p \wedge q \wedge \sim r$	$\sim p \vee \sim q \vee r$
T	F	T	$p \wedge \sim q \wedge r$	$\sim p \vee q \vee \sim r$
T	F	F	$p \wedge \sim q \wedge \sim r$	$\sim p \vee q \vee r$
F	T	T	$\sim p \wedge q \wedge r$	$p \vee \sim q \vee \sim r$
F	T	F	$\sim p \wedge q \wedge \sim r$	$p \vee \sim q \vee r$
F	F	T	$\sim p \wedge \sim q \wedge r$	$p \vee q \vee \sim r$
F	F	F	$\sim p \wedge \sim q \wedge \sim r$	$p \vee q \vee r$

Q) Find the DNF and CNF of $q \vee [p \wedge q] \wedge \sim[(p \vee r) \wedge q]$

DNF: $q \vee [p \wedge q] \wedge \sim[(p \vee r) \wedge q]$

$q \wedge [\sim(p \vee r) \vee \sim q]$

$q \wedge [(\sim p \wedge \sim r) \vee \sim q]$

$[q \wedge \sim p \wedge \sim r] \vee [q \wedge \sim q]$

(A) CNF: $q \vee [p \wedge q] \wedge \sim[(p \vee r) \wedge q]$

$= [(q \vee p) \wedge (q \vee q)] \wedge [\sim(p \vee r) \vee \sim q]$

$= [(q \vee p) \wedge q] \wedge [(\sim p \wedge \sim r) \vee \sim q]$

$= p \wedge (\sim p \vee \sim q) \wedge (\sim r \vee \sim q)$ (absorbtion Law)

Q Obtain DNF and CNF for $(p \rightarrow q) \wedge [\sim p \wedge q]$

(A) CNF
~~DNF~~: $(p \rightarrow q) \wedge (\sim p \wedge q)$

$(\sim p \vee q) \wedge (\sim p \wedge q)$

$(\sim p \vee q) \wedge (\sim p) \wedge (\sim p \vee q) \wedge (q)$ To Do!!

~~$(\sim p \vee q) \wedge \sim(p \vee \sim q)$~~

(v) DNF: $(p \rightarrow q) \wedge (\sim p \wedge q)$

$(\sim p \vee q) \wedge (\sim p \wedge q)$

$[\sim p \wedge (\sim p \wedge q)] \vee [q \wedge (\sim p \wedge q)]$

$(\sim p \wedge q) \vee [(q \wedge \sim p) \wedge [q \wedge q]]$

$(\sim p \wedge q) \vee [(q \wedge \sim p) \wedge q]$

(H.W) Obtain DNF of $p \wedge (p \Rightarrow q)$

Obtain CNF of $\sim(p \rightarrow (q \wedge r))$

18/03/24

Working Rule to ~~Objaan~~ Obtain PDNF:

- write the given statement in terms of $\vee, \wedge, \sim$ alone.
- Apply each term "$\wedge T$" ($p \wedge T \equiv p$
  $p \wedge \sim p \equiv T$)
- Insted of T, apply $p \vee \sim p$.
- Apply distribution law.
- Apply commutative law.

Working Rule To Obtain PCNF:

- write the given statement in terms of $\vee, \wedge, \sim$ alone.
- Apply for each term "$\vee F$" ($p \vee F \equiv p$
  $p \wedge \sim p \equiv F$)
- Instead of F, apply $p \wedge \sim p$.
- Apply distribution law.
- Apply commutative law.

Note: . If $\sim$ applied on entire problem, we use demorgans law.

- Remove tortology terms (T, F) i.e, maximum terms repetition is not possible.
- If any proposition is missing, we have to add it.
- Identical repeted terms need to be written only once.

Find PCNF by (i) Truth Table for $p \leftrightarrow q$.
(ii) without Truth Table.

i) With Truth Table:

($\wedge$) Min Terms – PDNF
($\vee$) Max Term PCNF

p	q	$p \leftrightarrow q$	
T	T	T	(PDNF)
T	F	F	(PCNF)
F	T	F	(PCNF)
F	F	T	(PDNF)

Max Term

$\sim p \vee \sim q$

$\sim p \vee q$

$p \vee \sim q$

$p \vee q$

$(\sim p \vee q) \wedge (p \vee \sim q)$ be the Max terms.

ii) without Truth Table:

$= p \leftrightarrow q$

$= (p \rightarrow q) \wedge (q \rightarrow p)$

$= (\sim p \vee q) \wedge (\sim q \vee p)$

$= (\sim p \vee q) \wedge (p \vee \sim q)$ ($\because$ Commutative law)

Find PCNF of $(\sim p \rightarrow r) \wedge (q \leftrightarrow p)$ with and without Truth Table.

i) with Truth Table:

p	q	r	$\sim p$	$\sim p \rightarrow r$	$q \leftrightarrow p$	$(\sim p \rightarrow r) \wedge (q \leftrightarrow p)$
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	T	F	F
T	F	F	F	T	F	F
F	T	T	T	T	F	F
F	T	F	T	F	F	F
F	F	T	T	T	T	T
F	F	F	T	F	T	F

$\therefore (\sim p \vee q \vee \sim r) \wedge (\sim p \vee q \vee r) \wedge (p \vee \sim q \vee \sim r) \wedge (p \vee \sim q \vee r) \wedge (p \vee q \vee r)$

be the PCNF.

ii) without Truth Table:

$$\equiv (\sim p \to r) \wedge (q \leftrightarrow p)$$

$$= (p \vee r) \wedge [(q \to p) \wedge (p \to q)]$$

$$= (p \vee r) \wedge [(\sim q \vee p) \wedge (\sim p \vee q)]$$

$$= (p \vee r \vee F) \wedge [(\sim q \vee p \vee F) \wedge (\sim p \vee q \vee F)]$$

$$= p \vee r \vee (q \wedge \sim q) \wedge [\sim q \vee p \vee (r \wedge \sim r)] \wedge [\sim p \vee q \vee (r \wedge \sim r)]$$

$$= (p \vee q \vee r) \wedge \underline{(p \vee \sim q \vee r)} \wedge (p \vee \sim q \vee r) \wedge (p \vee \sim q \vee \sim r)$$

$$\wedge (\sim p \vee q \vee r) \wedge (\sim p \vee q \vee \sim r)$$

$\therefore (p \vee q \vee r) \wedge \underline{(p \vee \sim q \vee r)} \wedge (p \vee \sim q \vee \sim r) \wedge (\sim p \vee q \vee r) \wedge (\sim p \vee q \vee \sim r)$

is the PCNF.

H.W

Find PDNF of $\sim p \vee q$ with and without Truth Table.

i)

p	q	$\sim p \vee q$	MinTerms
T	T	T	$p \wedge q$
T	F	F	
F	T	T	$\sim p \wedge q$
F	T	T	$\sim p \wedge \sim q$

$(p \wedge q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$

ii)

$\sim p \vee q$

$(\sim p \wedge T) \vee (q \wedge T)$

$(\sim p \wedge (q \vee \sim q)) \vee (q \wedge (p \vee \sim p))$

$(\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (q \wedge p) \vee (q \wedge \sim p)$

$(p \wedge q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$

26/3/24

H.W Find PCNF and PDNF of $(\sim p \to q) \wedge (p \leftrightarrow q)$ with and without Truth Table.

**

Rules Of Inference:

Argument: An argument in prepositional logic is a sequence of propositions. All ~~about~~ but the final proposition in the argument are called ~~pr~~ premises. And the final preposition is called the conclussion.

An argument is valid if the both or its premises implies that the conlsion is true.

i.e, $H_1, H_2, H_3, \dots, H_n$ are proposition / statements.

another c proposition

$$\underbrace{H_1 \wedge H_2 \wedge H_3 \wedge \dots \wedge H_n}_{\text{premises}} \Rightarrow \underbrace{C}_{\text{conclusion}}$$

If c follows $H_1, H_2, \dots, H_n$. Then c is valid conclusion.

If not, c is not valid. conclusion.

Steps are called valid arguments which are used.

Truth Table Technique: When "p" and "q" are two statements, then, "q" is said to logically follow "p." or "q" is a valid conclusion of the premises "p." if $p \to q$ is a toritology. Extending a ~~cot~~ conclusion is said to follow a set of premises $H_1, H_2, H_3, \dots, H_n$ if

$$H_1 \wedge H_2 \wedge H_3 \wedge \dots \wedge H_n \Rightarrow C$$

If a set of premises and a conclusion are given, it is possible to determine if conclusion follows the premisess by constructing relavent truth table as in the following questions.

1. $H_1: \sim p$, $H_2: p \vee q$, $C: q$

2. $H_1: p \to q$, $H_2: q$, $C: p$

p	q	$\sim p$	$p \vee q$	$p \to q$
T	T	F	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	T

i)

H_1	H_2	$H_1 \wedge H_2$	C
$\sim p$	$p \vee q$	$\sim p \wedge (p \vee q)$	q
F	T	F	T
F	T	F	F
T	T	T	T
T	F	F	F

$H_1 \wedge H_2 \neq C$

$\therefore$ C is not valid

H_1, H_2 are true in 3^{rd} row. i.e, C = True

ii)

H_1	H_2	$H_1 \wedge H_2$	C
$p \to q$	q	$(p \to q) \wedge q$	
T	T	T	T
F	F	F	T
T	T	T	F
T	F	F	F

$H_1 \wedge H_2 \neq C$

$\therefore$ C is not valid

H_1, H_2 are true in 1^{st} and 3^{rd} row.
C is not True in 3^{rd} row
C valid in 1^{st} row.

Hence It is not valid

- Types of Rules of Inference:

  Direct Proof:

  Indirect Proof:

  Conditional Proof:

Rule P: A premesis may be introduced at any step in the derivation.

Rule T: A formula may be introduced in the derivation.

Rules of Inference:

conjuctive $\rightarrow \wedge$

disjunctive $\rightarrow \vee$

- Modus Ponens: $p, p \rightarrow q \Rightarrow q$.
- Modus Tollens: $\sim q, p \rightarrow q \Rightarrow \sim p$
- Hypothetical Syllogism: $p \rightarrow q, q \rightarrow r \Rightarrow p \rightarrow r$
- Disjunctive Syllogism: $\sim p, p \vee q \Rightarrow q$
- Addition: $p \Rightarrow \dfrac{p}{p \vee q}$ or $\dfrac{q}{p \vee q}$
- Simplification: $p \wedge q \Rightarrow \dfrac{p \wedge q}{p}$ or $\dfrac{p \wedge q}{q}$

- i) Conjunction : $p, q \Rightarrow p \wedge q$

  ii) Disjunction : $p, q \Rightarrow p \vee q$.

- Delemma : $p \vee q,\ p \rightarrow R \Rightarrow R$

  $p \vee q,\ q \rightarrow R \Rightarrow R$

- $\sim(p \rightarrow q) \Rightarrow \sim q$
- $\sim p \Rightarrow p \rightarrow q$
- $q \Rightarrow p \rightarrow q$
- $\sim(p \rightarrow q) \Rightarrow p$.

Direct Proof : when a conclusion is derived from a set of premises ~~by~~ by using the equivalance and implication rule, then the process of derivation is called direct proof.

Indirect Proof : for ~~doing~~ doing indirect method, introduce the negation ($\sim$) of the conclusion as a additional premises and from the addition premesis together with the given premise, derive conclusion

Conditional Proof If a formula "S" can be derived from another formula "R" and a set of premises. Then the statement $R \rightarrow S$ can be derived from the set of premises tomorrow.

Problems on Direct Proof:

Show that $R \vee S$ follows logically from the premises

$$\underset{H_1}{C \vee D},\ \underset{H_2}{(C \vee D) \rightarrow \sim H},\ \underset{H_3}{(A \wedge \sim B) \rightarrow R \vee S},\ \underset{H_4}{\sim H \rightarrow (A \wedge \sim B)}$$

i) $C \vee D$ (Rule P)

ii) $C \vee D \rightarrow \sim H$ (Rule P)

iii) $\sim H$ (Rule T) (Modus Ponens: $p, p \rightarrow q \Rightarrow q$)

iv) $\sim H \rightarrow (A \wedge \sim B)$ (Rule P)

v) $(A \wedge \sim B)$ (Modus Ponen Rule T)

vi) $(A \wedge \sim B) \rightarrow R \vee S$ (Rule P)

vii) $R \vee S$ Rule T Modus Ponens.

$\therefore$ $R \vee S$ is following the given set of premises.

Show that $T \wedge S$ can be derived from the premises

$$\underset{H_1}{P \rightarrow Q},\ \underset{H_2}{Q \rightarrow \sim R},\ \underset{H_3}{R},\ \underset{H_4}{P \vee (T \wedge S)}$$

i) $P \rightarrow Q$ Rule P

ii) $Q \rightarrow \sim R$ Rule P $\qquad p \rightarrow q,\ q \rightarrow R \Rightarrow p \rightarrow R$

iii) $P \rightarrow \sim R$ Rule T (Hypothetical Syllogism)

iv) R Rule P

v) $\sim P$ Rule T $\qquad \sim q,\ p \rightarrow q \Rightarrow \sim p$ (Modus tollens)

vi) $p \vee (T \wedge S)$ Rule P

v) $T \wedge S$ Rule T $\qquad (\sim p,\ p \vee q \Rightarrow q$ Disjunctive Syllogism)

$T \wedge S$.

Show that $\sim P$ can be derived from the premises.

$(P \to Q) \wedge (R \to S)$, $(Q \to T) \wedge (S \to U)$, $\sim(T \wedge U)$, $P \to R$

i) $(P \to Q) \wedge (R \to S)$ (Rule P)

ii) $P \to Q$ (Rule T)

iii) $R \to S$ (Rule T)

iv) $(Q \to T) \wedge (S \to U)$ (Rule P)

v) $Q \to T$ (Rule T)

vi) $S \to U$ (Rule T)

vii) $P \to T$ Rule T 2,5 Hypothetical syll.

viii) $R \to U$ Rule T 3,6 "

ix) $P \to R$ Rule P

x) $P \to U$ Rule T 9,8

xi) $\sim(T \wedge U)$ Rule P

xii) $\sim T \vee \sim U$ Rule T

xiii) $\sim T \to \sim p$

xiv) $\sim U \to \sim p$

xv) $\sim P$

Demonstrate that R is a valid conclusion from the premises

~~P→Q~~ $P \to Q$, $Q \to R$, P

i) P Rule P

ii) $P \to Q$ Rule P

iii) Q Rule T (Modus Ponens)

iv) $Q \to R$ Rule P

v) R Rule T (Modus Ponens)

H.W

Show that $\sim P$ is a valid conclusion from the premises $\sim P \vee Q$, $\sim(Q \vee R)$, $\sim R$.

Problems on Indirect Proof :

i) Using indirect method of ~~proof~~ prove that

$P \to R, \; Q \to S, \; P \vee Q \Rightarrow S \vee R$

$H_1 \quad H_2 \quad H_3 \quad C$

Sol :-

i) $\sim S \wedge \sim R$ (Rule P additional Premise)

ii) $\sim S$ Rule T ① (Simplification $p \wedge q = q$, $p \wedge q = q$)

iii) $\sim R$ Rule T ①

iv) $P \to R$ Rule P

v) $\sim P$ (Rule T Modus Tollens) ③ ④

vi) $P \vee Q$ Rule P

vii) Q (Rule T Disjunctive Syllogism) ⑥

viii) $Q \to S$ Rule P

ix) S (Rule T Modus Ponens) ⑦⑧

x) $S \wedge \sim S$ Rule T ②,⑨

False

ii) Using indirect method ~~of~~ prove that

$P \to Q, \; Q \to R, \; P \vee R \Rightarrow R$

i) $\sim R$ Rule P (addition Premise)

ii) $Q \to R$ Rule P

iii) $\sim Q$ Rule T (①,② Modus Tollens)

iv) $P \to Q$ Rule P

v) $\sim P$ Rule T (③④ Modus Tollens)

vi) $P \vee R$ Rule P

vii) R Rule T (⑥ Disjunctive syllogism)

viii) $R \wedge \sim R$ Rule T ①,⑦

False.

Using indirect method show that $\underset{H_1}{R \rightarrow \sim Q}, \underset{H_2}{R \vee S}, \underset{H_3}{S \rightarrow \sim Q}, \underset{H_4}{P \rightarrow Q} \Rightarrow \underset{C}{\sim P}$

i) $\sim(\sim P)$ Rule P (additional Premise)

ii) P Rule T (double Negation)

iii) $P \rightarrow Q$ Rule P

iv) Q Rule T (Modes Ponens) 2 3

v) $R \rightarrow \sim Q$ Rule P

vi) $\sim R$ Rule T (Modes Tollens)

vii) $R \vee S$ Rule P

viii) S Rule T

ix) $S \rightarrow \sim Q$ Rule P

x) $\sim Q$ Rule T (Modes Ponens) 8, 9

xi) $Q \wedge \sim Q$ Rule T 4, 10

False

Conditional Proof (Rule CP) (Deduction Theorem)

Show that $R \rightarrow S$ can ~~the~~ be derived from the premises $P \rightarrow (Q \rightarrow S)$, $\sim R \vee P$ and Q.

i) R Rule P

ii) $\sim R \vee P$ Rule P

iii) P Rule T

iv) $P \rightarrow (Q \rightarrow S)$ Rule P

iv) $Q \rightarrow S$ Modes Ponens. Rule T

vi) Q Rule P

vii) S Rule T (Modes Ponens)

viii) $R \rightarrow S$ 1 7 Rule T

$R \rightarrow S$ be the valid conclusion.

Show that $P \rightarrow S$ can be derived from the Premises

$\sim P \vee Q$, $\sim Q \vee R$, $R \rightarrow S$.

i) P Rule P Additional Premise

ii) $\sim P \vee Q$ Rule P

iii) $P \rightarrow Q$ Rule T (Implication) 1 3

iv) Q Rule T (Modus Ponens)

v) $\sim Q \vee R$ Rule P

vi) $\cancel{R}\, Q \rightarrow R$ Rule T (Implication)

vii) R Rule T (Modus Ponens) 4 6

viii) $R \rightarrow S$ Rule P

ix) S Rule T (Modus Ponens) 7, 8

$P \rightarrow S$ Rule CP.

$\therefore$ $P\cancel{R} \rightarrow S$ is a valid conclusion.

1/4/24

Predicates And Quantifiers:

PREDICATE: A part of declerative sentence describing the property of an object or relation among the objects is called a predicate.

The logic based upon the analysis of predicate in any statement is called predicate logic.

Eg:- i) $\underbrace{X}_{\text{Subject}}$ $\underbrace{\text{is a good boy}}_{\text{Predicate.}}$

ii) $\underbrace{X}_{\text{Subject}} \underbrace{> 3}_{\text{Predicate.}}$

iii) $\underbrace{X+1}_{\text{Subject}} \underbrace{> x}_{\text{Predicate.}}$

iv) $\underbrace{\text{Ram}}_{\text{Subject}}$ $\underbrace{\text{is a batchler}}_{\text{Predicate.}}$

v) If $x = x$ then Truth value is

consider $p(x) : x > 3$ then

$p(4) : 4 > 3$ True

$p(2) : 2 > 3$ False

QUANTIFIERS: There are two types of quantifiers in the predicate calculus:
- Universal Quantifiers.
- Existential Quantifiers.

Universal Quantifier: The universal quantification of $p(x)$ is the statement of $p(x)$ for all values of x in the domain.

$\forall x \; p(x)$ denotes the universal quantification of $p(x)$.
(for every)

Here $\forall$ is called universal quantifier.

Example: i) Let $p(x): x+1 > x$ where the domain set of real numbers $\forall x\, p(x)$ is true.

ii) $p(x): x < 2$. where x is the set of integers $\forall x\, p(x)$ is false.

Existential Quantifiers; The ~~exential~~ existential quantifier quantification of $q(x)$ is the statement there exists an element x in the domain such that $q(x)$ is denoted by $\exists x\, q(x)$.

Here $\exists$ is called Exitential quantifier.

Eg:

i. Let $q(x): x > 3$. where domain is set of real numbers.

there ~~...~~ $\exists x\, q(x)$ is true.

ii) Let $q(x): x = x+1$ set of integers

i) There exists $\exists x\, q(x)$ is false

ii) for every $\forall x\, p(x)$ is True. ?? check.

Negation of Quantifiers:

Consider a statement

- Every student in our class has taken a course DM.

$p(x)$: x taken course in DM

$\forall x\, p(x) \Rightarrow$ Every student taken course DM.

$\sim(\forall x\, p(x)) \Rightarrow$ Non student has taken course in DM.

$\Rightarrow \exists x \sim p(x) \Rightarrow$

This is negation of statement.

Statement	Negation
$\forall x\, p(x)$	$\exists x \sim p(x)$
$\exists x\, p(x)$	$\forall x \sim p(x)$

Find Negation: Statement is

i) There is a honest Polition.

Solution:

$P(x)$: x is a honest Polition.

$\exists x\, p(x)$: There is a honest Polition.

Applying Negation:

$\sim[\exists x\, p(x)]$: $\forall x \sim p(x)$

$\forall x \sim p(x)$: All politions are not honest.

ii) All Americans ~~each~~ eat cheese and burgers.

$p(x)$: x eat cheese and burger

$\forall x\, p(x)$: All Americans eat cheese and burger.

Applying negation:

$\sim[\forall x\, p(x)]$: $\exists x \sim p(x)$

$\exists x \sim p(x)$: Some Americans don't eat cheese and burger.

iii) For every x, $x^2 > x$.

$\forall x\, (x^2 > x)$

Applying Negation: $\sim(\forall x\, (x^2 > x))$

$\exists x \sim(x^2 > x) \Rightarrow \exists x\, (x^2 < x)$.

iv) $\exists x\,(x^2 = 2)$

$= \exists x\,(x^2 = 2)$

$= \sim [\exists x\,(x^2 = 2)]$

$= \forall x\,\sim (x^2 = 2)$

$= \forall x\,(x^2 \neq 2)$.

Express the following sentences into mathanetical statements.

i) Every student in the class has studied calculus.

Let x: Student

~~$P(x)$: Student in the class studied calculus.~~

$S(x)$: x studied calcutus.

$C(x)$: x in class

$P(x)$: $S(x) \rightarrow C(x)$

$\forall x\,(S(x) \rightarrow C(x))$

ii) Some students in class visited Agra.

Let x be student.

$S(x)$: x in class $\quad C(x)$: x visited Agra.

$\exists x\,(S(x) \wedge C(x))$

iii) a) All lions are danigers animals.

b) Some lions dont drink coffee.

c) Some daingers animals don't drink coffee.

Let x is amimal

$L(x)$ = x is lion

$D(x)$: x is daingers.

$C(x)$: x drinks coffee.

a) $\forall x\,[L(x) \rightarrow D(x)]$

b) $\exists x\,[L(x) \wedge \sim C(x)]$

c) $\exists x\,[D(x) \wedge \sim C(x)]$

2/4/24

RULES OF INFERENSES for quantified Statements:

- Universal Specification: (US)

$$\forall x\, p(x) \Rightarrow p(c) \text{ for some } c.$$

- Universal Generalization: (UG)

$$p(c) = \forall x\, p(x) \text{ for any arbitary } c.$$

- Existential Specification: (ES)

$$\exists x\, p(x) = p(c) \text{ for some } c.$$

- Existential Generalization (EG)

$$p(c) = \exists x\, p(x) \text{ for any arbitary } c.$$

Verify the validity of the following arguments:

i)
→ Tigers are ~~dan~~ dangerous animals.

→ Those are tigers.

c ⇒ These are dangerous animals.

Let x be set of all animals

$T(x)$: x is a tiger.

$D(x)$: x is dangerous

H_1: $\forall x\, [T(x) \to D(x)]$

H_2: $\exists x\; T(x)$

C: $\exists x\; D(x)$.

1. $\forall x\, [T(x) \to D(x)]$ Rule P
2. $T(c) \to D(c)$ (US Rule T)
3. $\exists x\; T(x)$ Rule P
4. $T(c)$ (ES Rule T)
5. $D(c)$ 2, 4 Modus Ponens
6. $\exists x\; D(x)$ (EG)

These are dangerous animals.

ii)

- All integers are rational number.
- Some integers are powers of two.

C => Therefore, some rational numbers are power of two.

Let x be set of all numbers.

$I(x)$: x is integer.
$R(x)$: x is rational.
$P(x)$: x is power of two.

H_1 : $\forall x[I(x) \rightarrow R(x)]$
H_2 : $\exists x[P(x) \wedge I(x)]$
C : $\exists x[R(x) \wedge P(x)]$

① $\forall x[I(x) \rightarrow R(x)]$ Rule P

② $I(c) \rightarrow R(c)$ US Rule T

③ $\exists x[I(x) \wedge P(x)]$ Rule P

④ $I(c) \wedge P(c)$ ES Rule T

⑤ $I(c)$ Simplification Rule T

⑥ $R(c)$ ②⑤ Modus Ponens Rule T.

⑦ $P(c)$ ④ Simplification Rule T

⑧ $R(c) \wedge P(c)$ ⑥⑧ Conjuction Rule T

⑨ $\exists x[R(x) \wedge P(x)]$ (EG) Rule T.

Therefore, some rational numbers are power of two.

H.W

iii) · If I study ($P(x)$) I will not fail in exam ($Q(x)$)
· If I did not watch ($W(x)$) TV in evining I will study.
C => ~~If I didno~~ I must watch TV in the evening.